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Filtering Errors in LEO Trajectories Obtained by Kinematic GPS with Floated Ambiguities

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BIOGRAPHIES

Oscar L. Colombo has a first degree in Electrical Engineering from the National University of La Plata, in Argentina, and a PhD from the University of New South Wales, in Australia. He has worked on gravity field determination using satellite tracking data, precise orbit determination, and precise positioning with GPS. His collaboration with colleagues at the now Space Geodesy branch, Goddard Space Flight Center, dates back to the days of the first altimeter mission, GEOS-3.

Scott B. Luthcke received the B.S. degree in physics from the University of Maryland and the M.S. degree in applied physics from the Johns Hopkins University. He is currently working in the Space Geodesy Branch at NASA’s Goddard Space Flight Center. His research focuses on precision orbit determination and altimeter data analysis for planetary science applications. His interests include force and measurement modeling, and the analysis and reduction of a combination of data types including: GPS, TDRSS, SLR, DORIS, laser and radar altimetry.

David Rowlands received a BS and a MS in mathematics and a MS in geodetic Science from The Ohio State University. He has worked in the Laboratory for Terrestrial Physics at NASA's Goddard Space Flight Center since 1982. His primary focus has been on the development and implementation of measurement and force modeling algorithms for precision orbit determination.

Douglas S. Chinn received the BS and MEngr degrees (1973/1974) in engineering physics and the PhD degree (1982) in seismology from Cornell University. He has been working as a contractor for the Space Geodesy branch of Goddard Space Flight Center since 1984.

Susan Poulose received a MS in Physics from Kerala University, India and a MS in Medical Engineering from George Washington University. She has provided software development and data analysis support to scientists in Space Geodesy Branch of GSFC since 1978. Currently works for Raytheon ITSS.

ABSTRACT

We have developed and tested a way to precisely locate a low earth orbiter (LEO) carrying an on-board GPS receiver. The idea is to first use long-range kinematic GPS (lacking dynamic constraints), and then make an orbit fit (constrained by orbit dynamics) to the resulting trajectory, in order to filter out the kinematic errors. These errors tend to be rather large in satellite trajectories, because both the fast-changing subset of GPS spacecraft in view from the LEO, and the very long baselines between this and the fixed ground stations, together make ambiguity floating imprecise, and ambiguity fixing unreliable. The procedure outlined here requires a relatively small number of ground sites distributed around the world, selected from the much larger set of IGS stations on the basis of their consistently good performance. This method could be useful when processing altimetry and other satellite data requiring good geolocation. We have implemented this method at Goddard SFC using almost entirely pre-existing software. As an example, we have calculated two 24-hour orbit estimates of the oceanographic satellite TOPEX/
Poseidon, and another two for the satellite JASON. The resulting orbits agree to better than 5 cm RMS in height and 17 cm in three-dimensional RMS with the NASA Goddard Space Flight Center Precise Orbit Estimates (POE). Those POE, distributed in the case of TOPEX as part of the Geophysical Data Records, have been derived exclusively from DORIS Doppler and laser tracking data,

so they provide an entirely independent way to verify our GPS-based results.

INTRODUCTION

Satellite radar altimetry is a form of remote sensing from space where highly precise radar altimeters carried by spacecraft are used to map, primarily, the irregular shape of the mean sea surface and monitor its changes world-wide. This gives valuable information on the anomalies of the gravity field that hint at the structure of the solid crust and Earth's deep interior, as well as on ocean currents and changes in sea level associated with climatic processes and the interactions between ocean and atmosphere.
The interpretation of radar altimeter data requires a very good knowledge of the position of the altimeter, in particular its height above the Earth's surface. Therefore, precise orbit determination is a problem of the greatest importance for altimeter missions. The US-French mission TOPEX/Poseidon was the first one where a space qualified dual-frequency GPS receiver was used to obtain orbits to better than 10 cm in 3-dimensional precision, and better than 5 cm in height (Figure 1). This spacecraft carries one US and one French radio-altimeter, and the French Doppler Orbitography and Radiopositoning Integrated by Satellite (DORIS)) tracking system. It also has corner-cube retro-reflectors for satellite laser ranging (SLR), and an experimental GPS receiver.

Figure 1. The TOPEX/Poseidon oceanographic satellite. The large Earth-pointing radar altimeter dish is at the bottom. On top and at the rear, a long boom supports the GPS antenna. The high-gain communications antenna dish is on top the front section. For tracking, it also carries laser retro-reflectors (around the altimeter dish) and a DORIS Doppler receiver (small antenna pointing downwards).

This US-French cooperation still continues with JASON-1, another oceanographic mission with a satellite that carries radar altimeters among its sensors, and has DORIS, GPS and laser tracking. It has been placed on the same orbit as TOPEX/Poseidon, still in operation. At present, one follows the other above the same ground-track (Figure 2), passing over the same point on Earth about one minute apart.

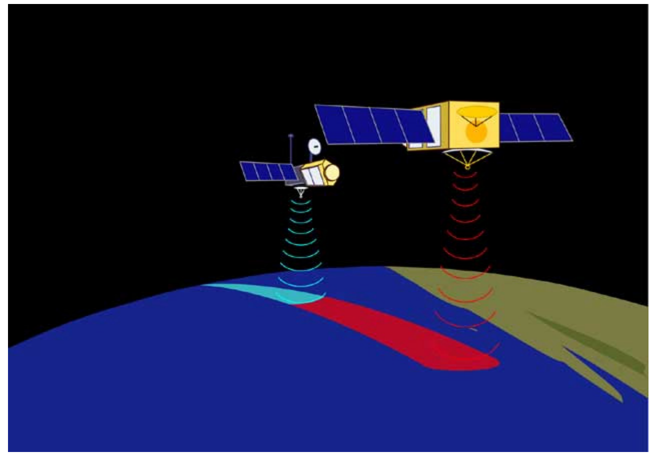

Figure 2. TOPEX/Poseidon and Jason-1 (nearest one in this picture), in their present co-orbiting configuration. The mean surface height above the reference ellipsoid is measured along a swath the width of the radar altimeter “footprint” (several km), by timing the return time of the radar pulses, and by precisely determining the orbit of each spacecraft to know its height.

The near-circular common orbit has a mean height of 1336 km, to reduce the effect of air drag. The inclination is 66 degrees, providing global coverage of all main ice-free bodies of water. The ground-track is repeated almost exactly every 9.9 days.
During the 80’s and early 90’s, when the TOPEX mission was being planned, the techniques for very precise (geodetic) positioning with GPS carrier-phase were just beginning to be tested and refined. The work directed towards getting suitable orbits for TOPEX/Poseidon using laser and radio tracking, also brought about the further development and maturation of the techniques used to this day for determining orbits to high precision with GPS [1]. Many of those involved in these activities also participated in NASA's Crustal Dynamics Project, perfecting static differential GPS positioning over very long baselines. Many of the basic ideas were the same for orbit and baseline determination, and their development clearly benefited from this overlap. In more recent times, the same ideas have been used to make possible sub-decimeter, long-range kinematic positioning (e.g., [2]).
At present, the orbits of satellite altimeters can be calculated to better than 10 cm in 3-dimensional position, and 1-3 cm rms in height. This can be achieved using GPS data, or else using satellite laser ranging (SLR), combined with Doppler radar. Either way, the orbits obtained are comparable in precision, and since the data used to determine them are entirely different, it is possible to use the SLR/DORIS-derived orbits to check the GPS ones, and vice-versa.

The method of choice for precise orbit determination using GPS carrier-phase data is the adjustment of numerical, dynamically integrated orbits, to fit the data. The procedure is based on Newton's laws of motion, where the various accelerations acting on the spacecraft (gravitation, atmospheric drag, radiation pressure, heat radiation recoil, etc.) are carefully calculated using precise models, and then numerically integrated in time to obtain position and velocity at discrete intervals. This is possible because, for the most part, spacecraft orbits are highly predictable, and spacecraft dynamics have been characterized extremely well, the result of many years of work conducted as part of the various national and international space programs.

This is exactly the opposite of what happens with most terrestrial vehicles (aircraft, ships, cars, etc.), whose dynamics are complex and, usually, poorly understood. In consequence, the method of choice for positioning terrestrial vehicles with GPS has been kinematic, which means relying entirely on the geometric strength of GPS, making no assumptions as to the forces that shape the trajectories.

Differential, kinematic positioning based on dual-frequency carrier-phase, the most precise form of GPS navigation, requires the removal of the phase ambiguities, or the precise estimate, as real-valued unknowns, of the corresponding biases Bc in the ionosphere-free linear combination Lc. The later approach is also known as “floating the ambiguities”, and is often the only way of dealing with ambiguities over very long baselines between roving and reference receivers. This approach works quite well, provided that there is enough data, distributed regularly over a long enough period of time, to estimate the Bc properly, along with other nuisance unknowns (residual tropospheric refraction, GPS orbit errors), and simultaneously with the instantaneous position of the rover. This is also known as a geodetic solution, or a grand solution.

When determining the trajectory of a satellite, the baselines are always long, since they cannot be any shorter than the LEO's height. So ambiguities are normally floated. This is no problem with dynamic solutions, because of the strong dynamic constraints, but it is the main factor limiting the precision of kinematic solutions. The very high velocity of the LEO (6-7 km/s) means that the GPS satellites in view keep changing rapidly, and the time for collecting data for any given double-difference is typically only 15-20 minutes. One way to increase the amount of data, and therefore the precision of the results, is to use many ground stations simultaneously. In one approach [3], as many as 100 IGS sites have been used. Further, one could use additional information to constrain the solution and increase its accuracy. One way to do this is to resolve at least a few ambiguities as exact integers (the additional information is that, in fact, ambiguities come in whole cycles). A different type of constraint, based on the smoothness of the orbit, is explained in this paper.

THE COMBINED TECHNIQUE

Rationale

For practical reasons, we prefer to work with a smaller set of ground stations, consisting of the IGS sites with the best records for reliable performance. We also would like to use software already available, with only minor changes and additions. Finally, we would like to have procedures easy to automate, so they can be used, eventually, in turnkey fashion.

Our basic approach to achieving all of the above, is to obtain first a kinematic trajectory, of limited precision, and then make a dynamic orbit fit to this trajectory. The errors in the kinematic solution have a typical duration of 10-15 minutes, same as the double differences and their poorly estimated Bc biases. This is short compared to orbit periods of 90-120 minutes. The fitted orbit is dynamically too rigid to follow in detail the trajectory at frequencies much higher than the orbital frequency. So the orbit-fitting process acts as a sharp low-pass filter that can remove much of the higher-frequency kinematic error. The result is a trajectory more precise than the original kinematic solution. Precise enough, as our results indicate, to be used, at least, as a “rapid science” orbit: Good enough for making a preliminary analysis of altimetry and other sensor data, before the final, more precise orbits are produced and distributed. Also, an integrated orbit can be easily calculated at the actual times when it is needed, and not only at those when enough GPS observations are available to form three or more double differences and get a kinematic fix on position. Finally, the kinematic trajectory to which the orbit is fitted, being free of any dynamic assumptions, can be used to check the dynamic orbit for blunders as well as more subtle errors in the force models, reference frame implementation, etc., used for its numerical integration.

In order to assess the quality of the end product, the fitted orbits, we use precise orbit estimates (POE) based on laser and Doppler tracking, totally independent of GPS and produced independently of our solutions. Furthermore, when available, we use laser data to verify our orbits, by comparing the laser-measured distances to the LEO with those calculated with the coordinates of the lasers and the orbits.

Outline of the Method

These are the main steps:

(a) A preliminary determination of the position and clock error of the LEO is made, using only the pseudo-range data from its GPS receiver, and the clock corrections and broadcast ephemeris from the Navigation Message. This means solving, at each epoch, a set of simultaneous quadratic equations (one for each pseudo-range measurement) representing intersecting spheres in 4-space, with four unknowns. In this way, the orbit determination

process can start without any previous knowledge of the orbit.
(b) Using the trajectory obtained by the non-linear, single-receiver procedure (a), the problem is linearized, and a first kinematic solution is obtained by sequential least squares (with the navigation Kalman filter), using double-differenced, dual-frequency pseudo-range from the ground sites and the LEO.
(c) A preliminary orbit is dynamically fitted to the pseudo-range solution (b).
(d) The preliminary dynamic orbit from (c) is used to help find and correct cycle slips in the double-differenced phase data from stations and LEO (movement along the orbit is smooth, so sudden jumps in the data away from the orbit indicate possible cycle slips).
(e) A kinematic solution is made using the double-differenced phase, corrected for cycle slips in (d), and the dynamic orbit fit as the a priori trajectory. Some constraints may be imposed on this solution to make it better, as described in the next section.
(f) A new orbit is fitted to the phase-based kinematic trajectory. The precision of this orbit is checked, by calculating the distances between the LEO and some laser-ranging sites, and comparing these distances to the actual laser ranges. If necessary, (e) and (f) may be iterated to improve the orbit. (The a priori uncertainties of the unknowns are not iterated.)
The process is illustrated with the plots in Figures 3-6.

Discussion:
One reason for iterating (e) and (f) is that the use of additional constraints in (e), however beneficial, may produce a kinematic solution that follows rather closely its a priori trajectory, the previous orbit fit. The kinematic solution is, nevertheless, better than the previous one, from either (c) or (e), so repeating the procedure is likely to result in a new fitted orbit closer to the true one. We found that the first two iterations brought about most of the improvement.
In the future, a less conservative editing of the data, that leaves more observations available for the kinematic solution, might make iteration unnecessary, by diminishing the influence of the a priori constraints.

"Truth" Orbits

To verify the quality of our results, we have used the very precisely and independently estimated orbits already available for both satellites.
For nearly a decade, the TOPEX/Poseidon mission's precise orbits have been computed at NASA Goddard Spaceflight Center's (GSFC) Space Geodesy Branch. Both before and after the launch of this mission, an extensive research activity was undertaken to develop and fine-tune the state of the art gravity and non-gravitational force models that are needed to support the precision orbit determination (POD) process [4], [5].
In addition, significant efforts to both validate software and algorithms and to study the orbit error characteristics have been conducted [6]. A number of metrics are used to gauge the performance of the resulting orbits including: orbit overlap comparisons, residual analysis, orbit intercomparisons between various institutions, data sub-set solution comparisons (e.g. orbits determined from SLR only compared to DORIS only solutions), and altimeter crossover residual analysis. Currently, the orbit performance for TOPEX/Poseidon can be characterized with a 2 cm radial RMS accuracy and sub-centimeter radial RMS precision over a 10-day repeat arc. JASON-1 precise ephemerides are also being computed at NASA / GSFC's Space Geodesy Branch in much the same manner as the TOPEX/Poseidon orbits. However, for JASON-1 POD we have near continuous data available from the dual frequency BlackJack GPS receiver. Therefore, in addition to SLR and DORIS Doppler-based orbit solutions, Goddard's Space Geodesy Branch has been computing reduced dynamic solutions based on GPS data alone. Comparisons between the GPS reduced dynamic solutions and the SLR+DORIS solutions show excellent agreement suggesting that for JASON-1 orbit performance is at the same level as for TOPEX/Poseidon even at this early stage in the mission, before model tuning and extensive analysis.

Dynamic Filtering of Kinematic Errors

The main reason for combining the kinematic and dynamic approaches is that the second one can be a very effective filter of errors from the first.
To understand why, one should look in some detail at the dynamics of a LEO orbit. Such orbits are usually near-circular. If one linearizes the equations of motion in a frame fixed to the orbit (with radial, along, and across-track axis that co-rotate with the orbit), making a "spherical Earth" approximation results in the so-called Hill's differential equations:

$$dC'' = a_C - n_o^2\, dC$$

$$dL'' = a_L - 2n_o\, dL'$$

$$dR'' = a_R + 3n_o^2\, dR + 2n_o\, dL'$$

where dR, dL, and dC are small perturbations (compared to the orbit radius) in the radial (R), along the orbit (L), and across the orbit plane (C) directions. Also: n_o is the orbital frequency (~ 10^{-3} rad/s), and a_C, a_L, a_R are small perturbing accelerations (e.g., force model errors) in the R, L, C directions. These equations have some important, but simple analytical solutions that are very useful both for understanding the behavior of LEO orbits, and for estimating and filtering ephemerides errors, not just in the case of LEO's, but of the GPS satellites as well [7], [8].
Transforming the above differential equations to the frequency domain, we get the transfer functions between

sinusoidal perturbing accelerations and the resulting orbital perturbations. For example, the magnitudes ||...|| of two of these functions are given by:

$$||C(w)/A_C(w)|| = 1/(n_o^2-w^2)$$

$$||L(w)/A_R(w)|| = 2n_o/\{w[4n_o^2-(w^2+3n_o^2)]\}$$

where C(w), L(w), Ac(w) and $A_{R(w)}$ are the Fourier transforms of dC, dL, a_C, and a_R, respectively. Both tend to infinity as w => n_o, and $||L(w)/A_R(w)||$ also tends to infinity as w => 0.

Clearly, there is a resonance at the orbital frequency n_o, and another at zero frequency. So one may expect orbital perturbations to have most of their energy concentrated around zero and one cycle per revolution in the along-track and radial components (because they are dynamically coupled, they share the resonances). The across component only has one resonance at n_o. In fact, its equation is that of a simple harmonic oscillator, and it is completely de-coupled from the other two components. Such perturbations will tend to exhibit strong ringing at once per revolution, plus very gradual, low-degree polynomial fluctuations caused by the zero-frequency resonance. In dynamically fitted (i.e., adjusted) orbits, the same theory governs the behavior of the errors caused by incorrect initial conditions and imperfect or incomplete force models. Typically, their ringing tends to present a "bow-tie" or "butterfly" envelope that is often largest at the ends of the orbit arc. Figure 3 shows these characteristics clearly: the once per revolution ringing in dR, dL, dC, a clear bow-tie envelope in dC, and an underlying slow polynomial fluctuation in dL.

The various parameters that are usually adjusted to make the orbit fit the data (as described in the following section), can correct the initial orbit estimate by subtracting from it the kind of resonant pattern shown here. <u>They cannot, therefore, change the orbit to follow the much faster errors that dominate the kinematic solution</u> (largely due to poor Lc bias estimates, caused by too short observing times). So the adverse effects of those errors on the fitted orbit tend to be smaller, and of lower frequency content, that is to say, smoother. In fact, our results show that this filtering action is so effective, that errors of up to several meters in the pseudo-range-only kinematic trajectory do cause errors of only a few decimeters in the dynamic orbit fit. (One should keep in mind that the points of a trajectory are fully three-dimensional data and, as such, intrinsically stronger than most tracking data.)

One remarkable property of the dynamic orbits obtained with this method is the much higher precision in the radial (or height) direction compared to that along or across-track. The reason for this is in the nature of orbit dynamics. The integrated orbits closely obey Kepler's Laws, particularly the 3/2 power law relating the semi major axis to the orbit period, and the law relating position and time to the eccentricity (equal areas swept in equal times). Both the semi-major axis and the eccentricity determine the amplitude of the oscillations in the height error. Because of the Laws, all points in the LEO kinematic trajectory, with their precisely timed positions, put a strong constraint on the size of the radial error. No such timing constraints exist on dL or dC.

DATA ANALYSIS

Kinematic procedure

The kinematic results shown in this paper were obtained by sequential processing of the carrier phase with a Kalman filter. Depending on the day, from 20 to 25 IGS stations were used as reference sites. In the long-baseline solutions, the observations were double-differenced between the rover and each reference receiver, and combined to form the ionosphere-free observable Lc. The Lc biases Bc (a linear combination of the L1 and L2 integer ambiguities) were estimated as real numbers (i.e. "floated"), along with other nuisance unknowns. The long-range kinematic software used [9], [10] allows the simultaneous determination of the following unknowns: (a) Corrections to the vehicle *a priori* known position (treated as three "white noise" states, with a 100 m *a priori* one-sigma precision per coordinate). (b) The Bc (treated as constants, each with a 10m *a priori* sigma). (c) Error in calculated tropospheric correction at each site (a constant plus a slow random walk error in the zenith delay). (d) GPS satellite orbit errors, as pseudo-initial state errors plus small (10^{-8} m/s^2) acceleration errors, using analytical orbit perturbation partials. (e) Error in reference station coordinates. In this case, neither the GPS orbits nor the IGS site coordinates were adjusted. We used the published, precise coordinates for the sites, corrected for tectonic motion and solid earth tides, and the final GPS orbits from the IGS (SP3) for the days in question, except for the one day in 1993, with TOPEX, for which we used the JPL orbits. Moreover, the dyamically fitted LEO orbits were converted into GPS antenna center trajectories using the center of mass/antenna center offset and the spacecraft orientation information (available in quaternions form). Altogether, about 50 error states were estimated in each solution. (To save computations and memory, orbit and bias states no longer active, are "recycled", yielding their places to newly active ones). Compressing data into 2-minute averages shortens calculations and facilitates the use of the mean height constraint described below. The kinematic results were obtained with software developed by the first author. It runs under UNIX, LINUX, FreeBSD, Windows 95, 98, ME, NT, and 2000.

Speeding up the Convergence of the Navigation Kalman Filter

The Kalman filter has to assimilate enough data to converge to a precise solution. The time needed for this should be kept as short as possible. Due to the very fast motion of the LEO, the typical double difference is formed with data from satellites that stay in common view of the ground sites and the orbiting receiver for only 15-20 minutes. While clearly needed in real time, fast convergence is always desirable. Even in post-processing, frequent gaps in GPS reception may cause the filter to be re-initialized too often, preventing its proper convergence, and resulting in a filter/smoother solution that is not precise enough. (The final level of precision achieved with the filter is also that of the post-processed trajectory calculated with the smoother.)

In making a kinematic solution, one ignores the often poorly known dynamics of the vehicle. In certain cases (e.g., a craft floating on water), the use of a slow-varying mean height constraint can shorten the convergence transient without introducing unwarranted assumptions as to how the vehicle moves otherwise. The same is true of the errors in the satellite orbit fitted to a previous kinematic trajectory (the pseudo-range-based solution, or a phase-based solution from a previous iteration). The noise in the GPS data can be reduced through averaging, to make the solution more precise. In the long-range technique used here a constraint on the mean height is easy to implement, because it uses *data compression*, or averaging [2, ibid.]. (To speed up calculations and economize computer resources such as hard disk space for scratch files) So it is easy to create pseudo-observations of the form:

Mean height (estimated) - mean height (model)
= Error in model (constant + random walk) + noise.

The "model" is the known ellipsoidal height of the time-varying sea level at the vehicle, in the case of a ship or a buoy. For a LEO, however, this value is zero. A random walk has been chosen to represent the gradual change in height, or in height error, because it is very easy to implement, and its variation can be made more or less smooth by changing the sigma of the white system noise that drives it. The constant is the unknown initial value of the height error. Clearly, this type of constraint could be also applied to the along and across-track components of the orbit error. We have not done so in this study, since we have tried not to modify the pre-existing software. In any case, the use of a height constraint benefits the accuracy in all three dimensions, and can speed up considerably the convergence of the navigation filter [11].

The Dynamic Orbit Fit

For the dynamic orbit determination part we have used GEODYN, the main geodynamics and geodetic analysis software developed at Goddard Space Flight Center. We model the forces acting on the satellite with a fixed box-wing model for the effect of solar radiation and drag, and with the JGM3 gravity field model [12]. We also estimate the orbit six initial state components, as well as a few force-related parameters.

Those are: one drag coefficient every four hours (TOPEX) or eight hours (JASON), and one daily set of four or five acceleration parameters (along- and across-track amplitudes of the sine and cosine of the mean anomaly, and also a small constant acceleration across track in the JASON solutions). These unknowns represent the lumped effect of small forces not modeled, or modeled incorrectly [7, ibid.]. So, altogether, there are at most 16 unknowns to be solved for each 24-hour orbit arc. Of several choices of force parameters tried, this one produced the best results. This is the same set of unknowns used for determining the NASA Precise Orbit Estimates (POE) for TOPEX, and the precise orbits that we used as "truth" for JASON (except for an additional small constant acceleration across-track). These "truth" orbits are all based on laser ranging and DORIS Doppler, and are, therefore, quite independent of the GPS data and related solutions.

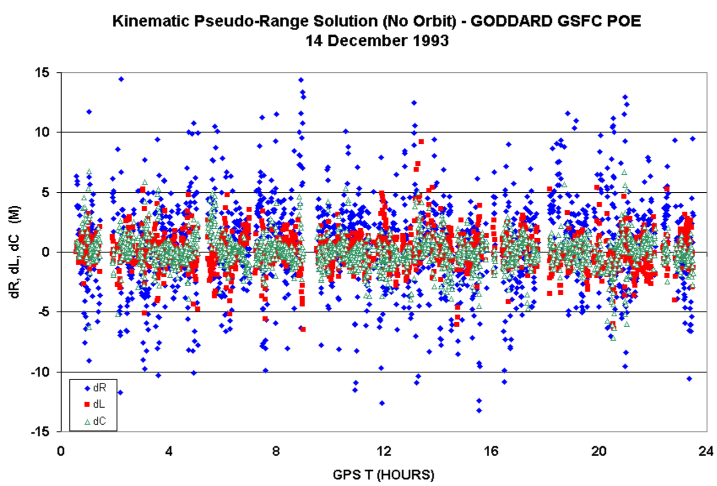

Figure 3. TOPEX: Outcome of steps (a) and (b): Kinematic solution with double-differenced pseudo-range (ionosphere-free combination). Shown here: differences dR, dL, dC (in m) between that solution and GSFC' s Precise Orbit Estimate (POE).

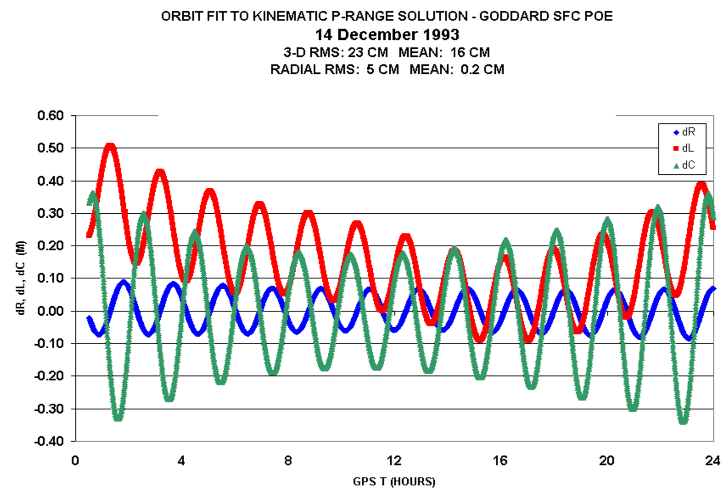

Figure 4. Step (c): Differences between orbit fitted to the pseudo-range kinematic solution of Figure 3, and the POE. Errors are more than 30 times smaller. Notice that dR (blue line) is much smaller than dL (red) and dC (green).

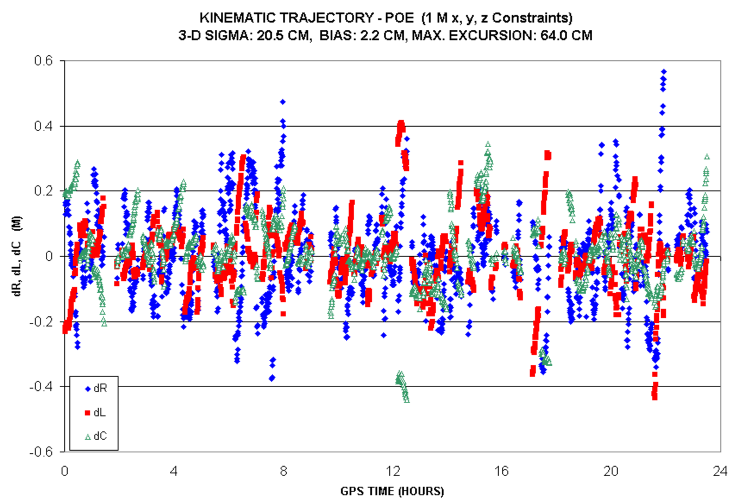

Figure 5. Steps (d) and (e): Comparison between the constrained kinematic phase-only (Lc) trajectory, and the POE.

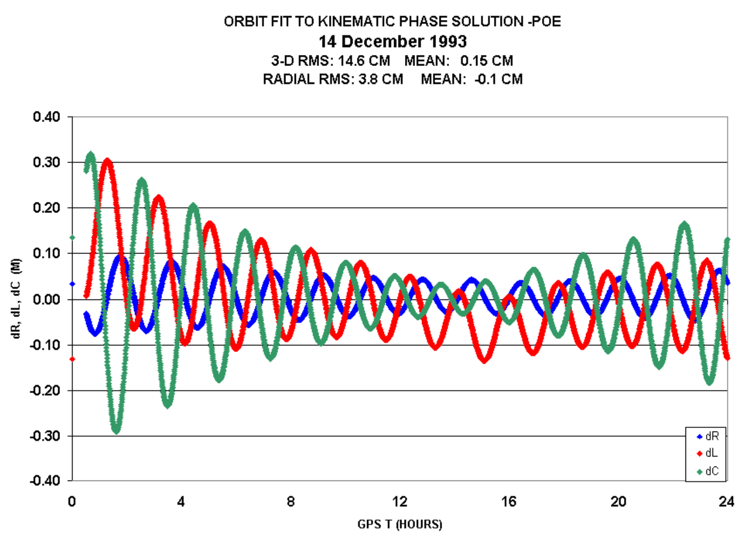

Figure 6. Step (f) Differences between the final dynamic orbit fit, and the POE.

RESULTS FOR TOPEX AND JASON

TOPEX Solutions

Two 24-hour solutions were made following the procedure outlined earlier on. Without any a priori knowledge of the orbit, first a pseudo-range-only kinematic solution was made. Then, a dynamic orbit fit to this solution, followed by a new, phase-only kinematic solution, constrained to have a slow-variation in height and, finally, a dynamic orbit fit to that phase-only solution. The results from each step are shown in figures 3-6 below. The agreement with the POE can be seen in Table1.

The errors in the pseudo-range-only solution (Figure 3) are strongly filtered by the dynamic orbit fit (Figure 4). This is particularly clear in the case of the radial component. The errors remaining after the dynamic orbit fit have the ringing behavior predicted by the near-circular orbit perturbation theory summed up in Hill's equations.

Figures 5-6 show patterns similar to those in Figures 3-4, although the size of the errors is much smaller. These later results are based, directly or indirectly, on the more precise carrier-phase data.

The following constraints were applied to the carrier-phase solution: dR was treated as a random walk with an initial uncertainty of 1 m, and a process noise of 8 cm/sqrt(10 minutes); dL and dC were treated as white-noise error states with 1 m uncertainty each (i.e., one standard deviation). The kinematic phase solution and the orbit fit steps were not iterated.

Twenty IGS sites were selected to form the ground network. The data from those sites and TOPEX were obtained in the form of RINEX files. The TOPEX data had been pre-edited at the Jet Propulsion laboratory (JPL), and their quality was better than that for most IGS sites. The data rate of the TOPEX receiver was once every 10 seconds, but the double-differences were formed once every 30 seconds, because that was the rate of the IGS sites. Elevation masks with a cutoff of 15 degrees above the horizon for the IGS sites, and of 10 degree for TOPEX, were used. The data main limitations were: Less than uniform distribution of IGS sites at the time, with a large gap in Asia and few stations in the Southern Hemisphere; and a maximum of only six satellites that could be tracked simultaneously by the TOPEX receiver.

The data were from 14 December 1993 and 24 June 1995, when Anti-Spoofing was off. (The TOPEX receiver cannot track L2 when the P-code is not available.)

The orbits were integrated dynamically in a quasi-inertial frame. The orbit is the trajectory of the satellite center of mass. It has to be converted to the trajectory of the GPS antenna in the Earth-fixed, Earth-Centered navigation frame implied by the IGS station coordinates and the JPL SP3 ephemerides of the GPS satellites. The vector offset between center of mass and antenna center is rotated from body frame to navigation frame using orientation quaternions from files provided by the TOPEX/Poseidon project. An opposite rotation and offset bring the kinematic trajectories back into the frame in which the fitted orbits were dynamically integrated.

JASON Solutions

Two 24-hours of data, for the 8 and 9 of March 2002, from 25 IGS stations and JASON, were analyzed in a very similar way as already explained for TOPEX.

One important departure was the use of laser ranging data to independently assess the quality of the dynamic orbit fits. The laser ranges were subtracted from the calculated distances between the satellite and the tracking sites, and the RMS of the differences was used as a measure of accuracy. This is listed in Table 2.

There are two GPS antennas on JASON. We used data from antenna GPS 2. The antennas are pointed 30 degrees off the vertical. To avoid poor reception, a 5-degree cutoff mask "above" the antenna plane was used, in addition to a 10-degree elevation mask above the Earth's limb. The elevation cutoff was chosen at 15 degrees for all the IGS sites. The calculations were carried out much as for TOPEX, except that there was a definite improvement after iterating twice the kinematic phase-only solution followed by a dynamic orbit fit.

Although JASON's receiver can track more satellites simultaneously than TOPEX's, the data is known to have glitches that may have caused excessive editing. At the same time, probably due to the increased level of ionospheric activity under near-solar maximum conditions, during days close to the equinox, the data from the IGS stations were rather heavily edited. Nowadays, there are procedures for editing JASON and IGS data more effective, less conservative, than those in our kinematic software. After upgrading the editor, in the near future, we plan to revisit these calculations. Our expectation is that, with more data left available after editing, results should improve significantly.

TABLE 1: TOPEX
Dparture of Dynamic Orbit Fit from GSFC's POE (Centimeters)

DAY	RSS	dR (RMS)	dR MEAN	Points in Orbit Fit
12/14/93	14.6	3.8	0.2	1801
06/24/95	8.6	1.4	0.2	1766

TABLE 2: JASON
Departure of Dynamic Orbit Fit from GSFC's POE (Centimeters)

DAY	RSS	dR RMS	dR MEAN	SLR RMS	Points in Orbit Fit
08/04/02	15.1	4.5	-0.3	9.9	1242
09/04/02	15.1	4.8	0.1	9.5	1367

CONCLUSIONS

We have tested a procedure that combines kinematic and dynamic techniques by calculating the orbits of the altimeter satellites TOPEX and JASON, estimating two 24-hour arcs for each. The kinematic solutions, started without prior knowledge of the orbit, were refined by dynamic orbit fits that filtered out much of the kinematic error. Those errors were caused, primarily, by imperfectly determined Lc phase biases, because of the very short times available for estimating them in the case of fast-moving LEO's.
The results of all four 24-hour solutions have been compared to precisely estimated orbits based on independent data (from SLR and DORIS).
The agreement with these orbits is equal or better than 17 cm in 3-dimensional RSS, and than 5 cm RMS in the radial (or height) component of the orbits.

The procedure has been designed to meet several conditions: It should use pre-existing kinematic and dynamic software, with little additional programming, to process already available data. The GPS ground data should be from a relatively small subset of IGS sites selected for their reliable performance. The method should be reasonably easy to develop into automated, turnkey-type operational software. The precision attainable should, at least, be good enough to produce rapid science orbits.
From our present results, we conclude that the procedure has met all the conditions that we have been able to test.

We expect that forthcoming upgrades to the data editor in the kinematic software should improve results significantly, by allowing more data into the solution than the more conservative procedures now in place, developed during the days when Selected Availability was still on.

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REFERENCES

[1] Bertiger, W.I., and 20 co-authors, "GPS Precise Tracking of TOPEX/POSEIDON: Results and Implications", Journal of Geophysical Research, Vol. 99, C12, Pp. 24,449-24,464, December 1994.

[2] Colombo, "O.L., Long Range Kinematic GPS", in *"GPS for Geodesy"*, 2nd Edition, A. Kleusberg and P. Teunissen, Editors. Springer-Verlag, 1998.

[3] Svehla, D., and M. Rothacher, "Kinematic Orbit of LEOs Based on Zero or Double-Difference Algorithms Using Simulated and Real SST Data", Proceedings 2001 Assembly of the IAG, Budapest, 2001.

[4] Nerem, R.S., Putney, B.H., Marshall, J.A., Lerch, F.J., Pavlis, E.C., Klosko, S.M., Luthcke, S.B., Patel, G.B., Williamson, R.G. and Zelensky, N.P., "Expected Orbit Determination Performance for the TOPEX/Poseidon Mission," IEEE Transactions on Geoscience and Remote Sensing, Vol. 31, No. 2, March 1993, pp. 333-354.

[5] Marshall, J.A. and S.B. Luthcke, "Nonconservative Force Model Performance for TOPEX/Poseidon Precision Orbit Determination," Journal of Astronautical Sciences, Vol. 41, No. 2, April-June 1994, pp. 229-246.

[6] Marshall, J.A., Zelensky, N.P., Luthcke, S.B., Rachlin, K.E., and Williamson, R.G., "The Temporal and Spatial Characteristics of TOEX/Poseidon Radial Orbit Error," Journal of Geophysical Research, Vol. 100, No. C12, Second TOPEX/Poseidon Special Issue, Dec. 1995, pp. 25,331-25,352.

[7] Colombo, O.L., "Ephemeris errors of GPS satellites", Bulletin Geodesique, Vol. 60, No. 1, Paris, 1986.

[8] Colombo, O.L., "The dynamics of GPS orbits and the determination of precise ephemerides", Journal of Geophysical Research, Vol. 94, B7, pp. 9167-9182, 1989

[9] Colombo, O.L., "Errors in Long Distance Kinematic GPS", *Proceedings of the ION GPS '91*, Albuquerque, N.M., September 1991.

[10] Colombo, O.L., and A.G. Evans, "Testing Decimeter-Level, Kinematic, Differential GPS Over Great Distances at Sea and on Land", *Proceedings ION GPS '98*, Nashville, Tennessee, September 1998.

[11] Colombo, O.L., Evans, A.G., Ando, M., Tadokoro, K., Sato, K., T. Yamada, "Speeding Up The Estimation Of Floated Ambiguities for Sub-Decimeter Kinematic Positioning at Sea", Proceedings ION GPS-2001, Salt Lake City, September 2001.

[12] Tapley, B.D., and 14 co-authors, "The Joint Gravity Model 3", Journal of Geophysical Research, Vol. 101, No. B12, December 1996.